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Bombardment Analysis Method for Assessing Truss System Combat Survivability

Hsing-Juin Lee*
National Chung-Hsing University, Taichung, Taiwan 40227, Republic of China

The system combat survivability of structures is an essential issue of design analysis for many important structures such as the truss boom of an attack helicopter. Its system failure due to combat damage may cost much more than the repairable nonsystem failure. Nevertheless, before the difficult system survivability of two-dimensional multiple-story and statically indeterminate truss structures can be assessed, the bombardment survivability of each truss component should be predicted somehow. In this article, the Poisson process is used to model the approaching situation of fragments from a high explosive warload detonated in the vicinity of the targeted area encompassing a truss structure. Then a judicious Lee-Buffon method is developed and merged with the Poisson's model to accommodate the bombardment analysis of truss components due to small incoming fragments in a decomposed and probabilistic sense. Moreover, the ingenious combinational pivotal decomposition method is employed to assess the truss system combat survivability systematically. With minor modification, this analysis scheme may also be applied for space station truss structures subject to micrometeoroid/debris impacts.

Introduction

PROBABILISTIC structural mechanics and system reliability have become active research areas in recent years as indicated by burgeoning publications and associated conferences. Some papers investigate combat survivability of general tactical aircraft,[1-3] defense helicopters,[4,5] or specifically airborne propulsion systems.[6] Also, a good general reference[7] on aircraft combat survivability has appeared. On the other hand, some authors evaluate the reliability of truss structures,[8-11] concern system reliability of structures,[12-15] or assess associated research direction and needs.[16,17] Nevertheless, due to its inherent difficult nature, the bombardment and system survivability of truss structures are not fully addressed in literatures.[18-20] From previous aircraft combat experience in large-scale wars and many smaller scale skirmishes, high combat attrition rate has often been learned. A combat aircraft shot down may cost 1000-fold more than its repair price tag if its structure remains uncollapsable after being damaged by gunfire. Consequently, the system survivability virtually becomes an essential issue of design analysis for combat aircraft and other strategically important structures. An ingenious combinational pivotal decomposition method (CPDM)[21] has been developed to establish a systematic and efficient way for assessing the system combat survivability of this type of structure. However, before such a CPDM method can be applied to analyze the truss system survivability, the bombardment analysis may have to be performed to predict the survivability of each truss component of the system after suffering a session of gunfire.

In this study, the bombardment analysis method for assessing the system survivability of two-dimensional, multiple-story, and statically indeterminate truss structures will be specially investigated. First, the Poisson process is used to model the approaching rate of fragments from a high explosive warload detonated in the vicinity of the targeted truss structure. Then an interesting decomposition technique is used to accommodate the bombardment analysis as a quasi-Buffon's falling-needle problem. Subsequently, the CPDM method can be used to assess the system survivability with known failure probability of each truss component through foregoing bombardment analysis. This methodology will be detailed in the following sections.

Lee-Buffon Bombardment Analysis Method for Truss Structures

Buffon's Needle-Dropping Problem

In the year of 1777 A.D., a French mathematician Conte de Buffon presented an interesting problem of randomly dropping a needle on a vast plane full of parallel lines with distance l which is larger than the needle length l_1. Subsequently, the probability of the needle hitting any one of the parallel lines was derived by considering the randomness in regard to the whereabouts of the needle center and angle of attitude with respect to the parallel lines as shown in Fig. 1. If the center of needle happens to have a distance h to a closest parallel line, then

$$\frac{l_1}{2}\cos\alpha \geq h \tag{1}$$

is the necessary and sufficient condition for the needle to contact this closest line. Therefore, the hitting probability $[2l_1/(\pi l)]$ can be easily obtained as the ratio of the half-cosine crosshatched area to the rectangular area $(\frac{1}{2}\pi l)$ as shown in Fig. 2.

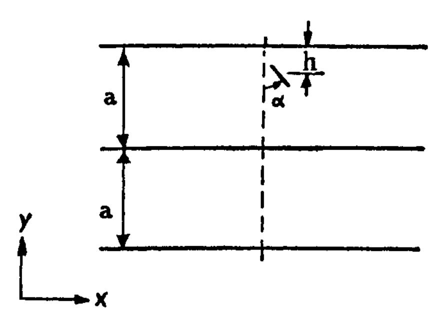

Fig. 1 Buffon's needle-dropping problem on vast plane with parallel lines.

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*Associate Professor, Department of Mechanical Engineering. Member AIAA.

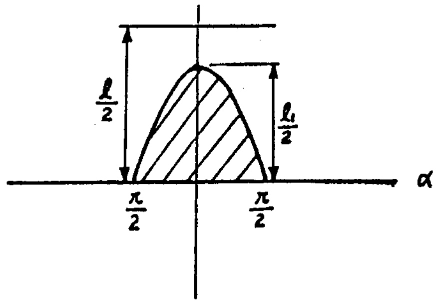

Fig. 2 Representative shape for needle-dropping hitting probability distribution.

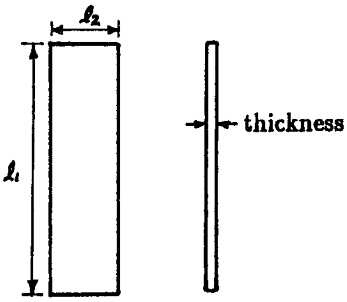

Fig. 3 Representative preformed rectangular fragment with thin thickness.

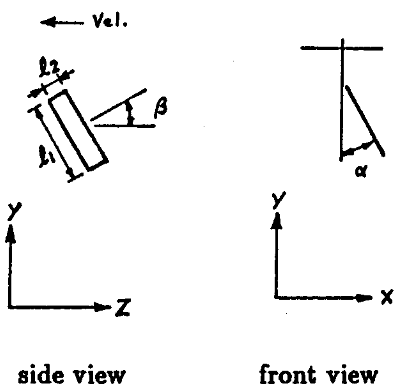

Fig. 4 Fragment approaching attitude toward targeted lines.

Bombardment Analysis

There are a variety of threats which might be thrown on typical combat aircraft, the hostile arsenal may range from machine guns to guided missiles. However, one of the most common kill modes is due to high explosive fused warhead detonated in the vicinity of the targeted combat aircraft by some target detection device and fuse logic regarding the relative motion between target and warhead. A typical example is the renowned air-to-air Sidewinder missile (AIM-9P-5) with laser proximity detection fuse. Upon detonation, the high-explosive warhead instantly breaks into hundreds or even thousands of high-speed spraying fragments, usually with particularly designed and preformed size and shape to optimize their attack effectiveness.[7] Some of these fragments may reach and penetrate the targeted structure, and therefore, damage or destroy it. In this study, the fragment is assumed in the form of a thin rectangular plate with relatively small length l_1 and width l_2 (Fig. 3), instead of the thin needle in the foregoing Buffon's problem. Generally, dimensions of fragment can be assumed much smaller than the length of targeted truss component (e.g., $\frac{1}{20}$ or smaller) for later accommodation of statistically independent combat failure probabilities between truss components. In order to avoid unnecessary complications and focus on presenting this novel analysis methodology, the approaching route of the fragment can first be assumed normal to the targeted X-Y plane, other variational cases can be easily considered later with minor modification of processing. Due to the rectification effect of aerodynamic force, the fragment can be imagined as a small rectangular blade slicing into the target plane, in other words, the plane of fragment is always normal to the target plane X-Y. Furthermore, for the purpose of more generality, the fragment is allowed to have an arbitrary angular attitude with respect to the X-Z plane as indicated by the angle β in Fig. 4, i.e., β is the random angle between the normal line of a fragment in the direction of width and the X-Z plane. This degree of freedom (DOF) is in addition to the arbitrary angular freedom of attitude α already owned by the Buffon's needle as shown in Fig. 1, the random variable α is the angle between the plane of incoming fragment and the Y-Z plane. Subsequently, similar to the solution scheme of Buffon's problem, with the easily satisfied assumption that

$$\sqrt{l_1^2 + l_2^2} < l[\cos(\pi/4)] \tag{2}$$

the following equation:

$$\left(\frac{l_1}{2}\cos\beta + \left|\frac{l_2}{2}\sin\beta\right|\right)\cos\alpha \geq h$$

for

$$\left(\alpha = -\frac{\pi}{2} \sim \frac{\pi}{2}\ \text{rad.}, \qquad \beta = -\frac{\pi}{2} \sim \frac{\pi}{2}\ \text{rad.}\right) \tag{3}$$

must be satisfied for the warhead fragment to hit any parallel lines. Therefore, referring to Fig. 5, the hitting probability can be expressed as

$$\frac{1}{\left(\frac{l}{2}\right)\left(\frac{\pi}{2}\right)^2}\int_{\alpha=0}^{(\pi/2)}\int_{\beta=0}^{(\pi/2)}\left(\frac{l_1}{2}\cos\beta + \frac{l_2}{2}\sin\beta\right) \times \cos\alpha\, d\alpha\, d\beta = \frac{4}{\pi^2}\left(\frac{l_1 + l_2}{l}\right) \tag{4}$$

Decomposition of Targeted Truss Structure into Buffon's Problem

Before the above-developed technique for assessing hitting probability can be appropriately applied, the representative

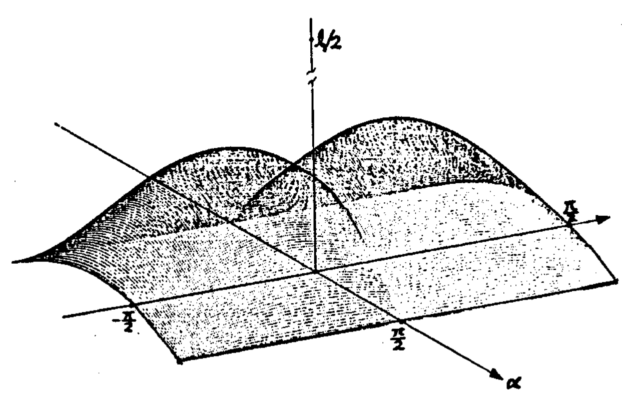

Fig. 5 Representative three-dimensional shape for hitting probability distribution of fragment on parallel lines.

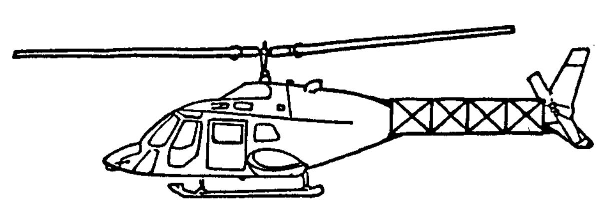

Fig. 6 Truss-simulated boom portion of a helicopter.

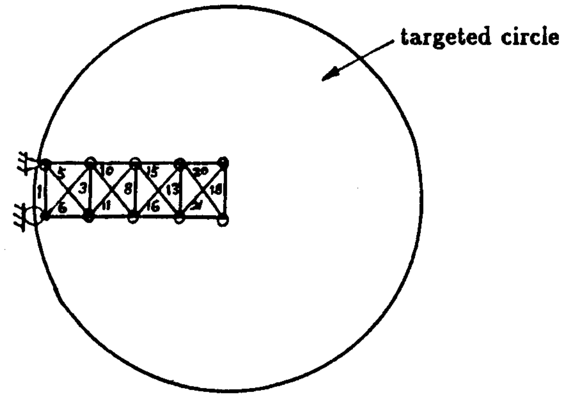

Fig. 7 Four-section truss structure enclosed in targeted circle.

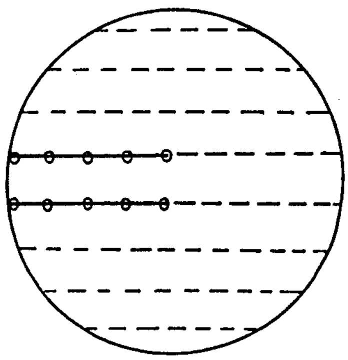

Fig. 8 Decomposed horizontal truss bars with complementary parallel dotted lines.

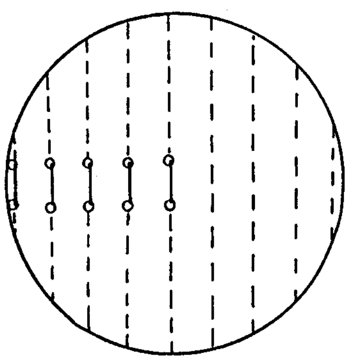

Fig. 9 Decomposed vertical truss bars with complementary dotted lines.

targeted two-dimensional statically indeterminate four-story truss (simulating the boom portion of a helicopter, Fig. 6) should be preprocessed in an interesting manner. Basically, the two-dimensional truss structure (Fig. 7) can be decomposed into four sets of parallel lines representing the center lines of truss components as shown in Figs. 8–10, where dotted complementary lines are added appropriately in the circular targeted area to accommodate the Buffon-type analysis. The target area is not necessary in a circular shape, in fact, any appropriate shapes such as a square or a rectangle can be utilized also, as long as the incoming fragments are statistically unbiased for their attacking points in the given target area. The size of the target area should be large enough compared with the fragment to well simulate the vast plane as in Buffon's needle-dropping problem in order to mitigate any undesirable boundary effect. Generally, this condition can be easily fulfilled satisfactorily due to the usual small size of a fragment. For example, taking Fig. 8, in consideration of the random nature of the process, the hitting probability on these solid lines can be calculated as

$$\frac{4(l_1 + l_2)}{\pi^2 l} \times \frac{8l}{L_1} = \frac{32(l_1 + l_2)l}{\pi^2 l L_1} \tag{5}$$

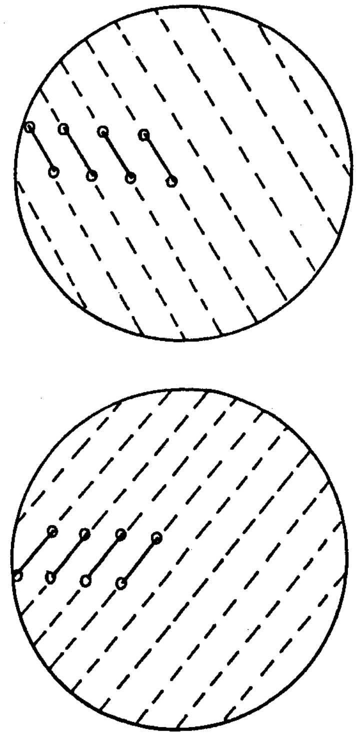

Fig. 10 Decomposed truss cross bars with complementary dotted lines.

Where L_1 is the summed length of all solid and dotted lines in Fig. 8, L_1 can be easily determined for the given target circle and truss. Also $8l$ is the summed length of the solid lines. In consideration of an unbiased hitting probability for any segments of lines in the target circle, Eq. (5) shows the appropriate share of hitting probability for solid lines in Fig. 8. Subsequently, for a single incoming fragment, the probability of hitting a particular horizontal truss component P_{h1} can be obtained by just dividing the above probability by the number of components constituting the solid line in Fig. 8:

$$P_{h1} = \frac{4(l_1 + l_2)}{\pi^2 L_1} \tag{6}$$

Which is also the hitting probability of each vertical truss component in Fig. 9. Likewise, the hitting probability of each set of crossbars in Fig. 10 can be expressed as

$$P_{h2} = \left(\frac{4(l_1 + l_2)}{\pi^2\{l[\cos(\pi/4)]\}}\right)\left\{\frac{\dfrac{l}{[\cos(\pi/4)]}}{\dfrac{L_1}{[\cos(\pi/4)]}}\right\} = \frac{4(l_1 + l_2)}{\pi^2 L_1[\cos(\pi/4)]} \tag{7}$$

Fragment Approaching Modeled as a Poisson Process

In reference to the renowned alpha rays bombardment experiment performed by atomic science forerunners Rutherford and Greiger in the year of 1910 A.D., the warhead fragment approaching behavior can be well modeled as a Poisson process accordingly. Other approaching models can be employed as usual, as long as the model used can appropriately address the approaching behavior of incoming fragments. With this understanding in mind, given an average approaching rate ν (number/second) of the warhead fragments to the targeted circle, then the probability of n fragments happening to hit the targeted circle during a transient combat session t second is designated as $P_H(N_t = n)$, which can be expressed as

$$P_H(N_t = n) = \frac{(\nu t)^n}{n!} e^{-\nu t}, \qquad n = 0, 1, 2, \ldots . \infty \tag{8}$$

Again, in order to avoid unnecessary complications and to highlight the analysis methodology itself first, it is assumed that a truss component will fail if hit by fragments at least once. Of course, other types of failure criteria may be easily applied as appropriate, depending on encountered situations, e.g., for smaller fragments it may be assumed that a truss component will fail if hit by any r out of k pieces of incoming fragments. Moreover, sometimes certain smaller effective impact-breaking dimensions instead of true fragment size can be wisely used in the above hitting equation to assure imparting sufficient fragment kinetic energy to targeted truss component. For a combat session of t seconds, if n fragments happen to hit the targeted circle, then the probability that a horizontal truss component may not survive the t-second attack can be expressed as

$$Q_{1,n} = 1 - (1 - P_{h1})^n \tag{9}$$

Where P_{h1} is from Eq. (6). While the incoming fragments are modeled as a Poisson's process as discussed above, instead of a deterministic number, the theorem of total probability can be used to assess the probability of failure P_{f1} of a horizontal truss component after endurance of a t-second bombardment session

$$P_{f1} = \sum_{n=1}^{\infty} \frac{(\nu t)^n}{n!} e^{-\nu t}[1 - (1 - P_{h1})^n] \tag{10}$$

For example, let ν be 300 fragment/s, and t be 1 s, then

$$P_{f1} = \sum_{n=1}^{\infty} \frac{(300)^n}{n!} e^{-300}[1 - (1 - P_{h1})^n] \tag{11}$$

Due to the extremely small contribution to the component failure probability, and nonproportionally extremely large amount of numerical calculation, it may be appropriate to cut off the high-end and low-end of the Poisson's probability mass function. Meanwhile, this cutoff of low-end will be helpful later in validating the assumption of statistically independent probabilities of failure between components in the truss system. The revised low-end n and high-end n may be taken as 60 and 1500, respectively, in consideration of the ν which is given as 300 fragment/s for the current case. From the Poisson's cumulative mass function for $n = 0 \sim 59$

$$P_{f1} = \sum_{n=0}^{59} \frac{(300)^n}{n!} e^{-300} = 6.52309 \times 10^{-65} \tag{12}$$

the low-end minimal contribution can be easily recognized. On the other hand, the high-end contribution for $n = 1501 \sim \infty$ is virtually approaching zero. As the above example demonstrates, usually it may be good enough to take the high-end as $5\nu t$ and the low-end as $0.2(\nu t)$. Moreover, νt is also the mean value of a Poisson's mass function, a higher νt will generally increase the component bombardment failure probability as Eq. (10) indicates. Thus, after the endurance of a combat session, the P_{f1} for any horizontal or vertical truss component can be closely expressed as

$$P_{f1} \approx \sum_{60}^{1500} \frac{(\nu t)^n}{n!} e^{-\nu t} \left\{1 - \left[1 - \frac{4(l_1 + l_2)}{\pi^2 L_1}\right]^n\right\} \tag{13}$$

Similarly, the P_{f2} for any crossbar of this truss structure can be written as

$$P_{f2} \approx \sum_{60}^{1500} \frac{(\nu t)^n}{n!} e^{-\nu t} \left\{1 - \left[1 - \frac{4(l_1 + l_2)}{\pi^2 L_1 \left(\cos \frac{\pi}{4}\right)}\right]^n\right\} \tag{14}$$

Through the above bombardment analysis, the probability of failure for each truss component has thus been obtained. For a nondimensional numerical example, let

$$l = 100, \quad l_1 = 5, \quad l_2 = 1, \quad L_1 = 10{,}000 \tag{15}$$

then, for 1-s bombarding with $\nu = 300$ fragment/s, P_{f1} and P_{f2} are respectively as follows:

$$P_{f1} = 0.07035, \qquad P_{f2} = 0.0887 \tag{16}$$

Combinational Pivotal Decomposition Method for Assessing Truss System Survivability

The system combat survivability of structures is an essential issue of design analysis for many important structures. After endurance of a session of gunfire bombardment, two-dimensional, multiple-story, and statically indeterminate truss structures can easily have millions or even trillions of possible damage modes.[21] The identification and pass/fail judgment of this gigantic number of modes present an insurmountable workload for existing analysis techniques, such as a raw fault tree analysis, a truth table analysis, or even the Monte Carlo sampling analysis. In that concern, an ingenious combinational pivotal decomposition method (CPDM)[21] has been developed to establish a systematic and efficient way for evaluating the system combat survivability of this type of structure. This method makes use of selected key bars in a truss structure as well as pivotal decomposition concept in an enhanced manner to circumvent the original dilemma.

Preliminary Combat Survivability Evaluation

Techniques to evaluate the combat survivability of two-dimensional one-story statically indeterminate truss structures will be briefly reviewed here. Fig. 11 shows such a single-story truss structure. Assuming that the statistically independent probability of failure for each truss member is given as P_f which depends on how fierce and long the bombardment has been proceeded as discussed in the above section. The combat fierceness can in turn depend on weaponry used, firepower actuated, and many other tactical factors involved. In general, P_f can be taken from empirical statistical data and/or the foregoing Lee-Buffon bombardment analysis. The system failure is defined as the appearance of any movable hinged node due to gunfire destruction of truss member(s) in the system, since it means disastrous truss system collapse and/or loss of its function as a truss structures with free-moving node(s). Additionally, assuming that as long as a combat structure does not become collapsable, it can sustain the design loads applied. Since the concurrence of both rare extreme events of severe combat damages and worst loading conditions (e.g., worst flight weather for an attack helicopter with damaged boom) has generally fairly low probability.

With the above system failure criteria in mind (e.g., Fig. 12), in case of the failure of truss members 3 and 4, node 4 will become movable, therefore this damage mode will constitute a system failure. Alternatively, as bar 5 and bar 6 fail, the original 6-bar truss structure becomes a standard 4-bar linkage mechanism, where nodes 3 and 4 become movable (Fig. 13). A close examination of the 6-bar truss system with its redundancy characteristics in mind reveals that any possible damage modes with failure of two or more truss members will constitute the system failure of this truss structure. Therefore,

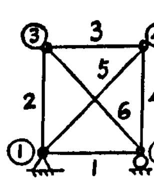

Fig. 11 Single-story truss structure.

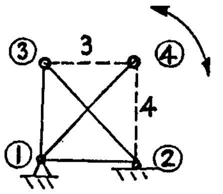

Fig. 12 Damage mode with bar 3 and bar 4 failed.

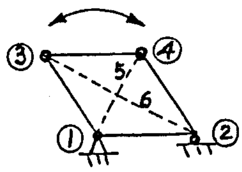

Fig. 13 Damage mode with bar 5 and bar 6 failed.

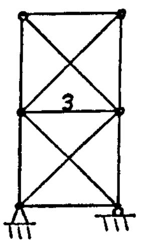

Fig. 14 Condition with bar 3 safe.

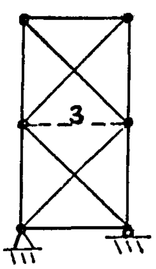

Fig. 15 Condition with bar 3 failed.

for this 6-bar truss structure, the system combat survivability can be immediately given as

$$(P_s)_{\text{syst}} = (1 - P_f)^6 + 6(1 - P_f)^5 P_f = P_s^6 + 6P_s^5 P_f \quad (17)$$

where $P_s = 1 - P_f$, it is the statistically independent probability of survival for each truss member. The first term in Eq. (17) is the probability for all six bars in safe condition, the second term gives the probability for six cases with one bar failed and the other five bars in safe condition, these are cases for the truss system to survive.

Furthermore, for the two-story 11-bar truss system shown in Fig. 14, the situation is somewhat different. There are more than 2000 ($2^{11} = 2048$) possible structure damage modes, some modes are catastrophic in the system sense, but others not. The preliminary pivotal decomposition technique can be conveniently used to evaluate the system combat survivability. First, it is necessary to recognize the special bar 3 which is seen to separate the upper and lower portion of this truss structure, this key bar is taken as the basic pass/fail "pivot." Bar 3 is either in a "pass" condition or in a "fail" condition as shown in Figs. 14 and 15, respectively. In the case of Fig. 14, both the upper and lower portion should survive for the system to survive, under this condition, these two portions can be handled separately in calculating the system combat survivability as follows:

$$({}_pP_s)_{\text{syst}} = P_s(P_s^5 + 5P_s^4P_f)^2 \quad (18)$$

Where presubscript p in $({}_pP_s)_{\text{syst}}$ indicates the survival of key bar 3. The squared term in Eq. (18) means both the upper and lower portions separated by the pivotal bar should be safe for the system to survive. In the case of Fig. 15, the $({}_fP_s)_{\text{syst}}$ becomes

$$({}_fP_s)_{\text{syst}} = P_f(P_s^{10} + 10P_s^9P_f) \quad (19)$$

It means that with the pivotal bar failed, the remainder cannot lose more than one bar for the system to survive. This assertion makes use of the mechanism characteristics that any two or more breakage of bars among the 10 remaining bars in Fig. 15 will constitute a system failure configuration. Together, the system combat survivability is

$$(P_s)_{\text{syst}} = ({}_pP_s)_{\text{syst}} + ({}_fP_s)_{\text{syst}} = P_s(P_s^5 + 5P_s^4P_f)^2 + P_f(P_s^{10} + 10P_s^9P_f) \quad (20)$$

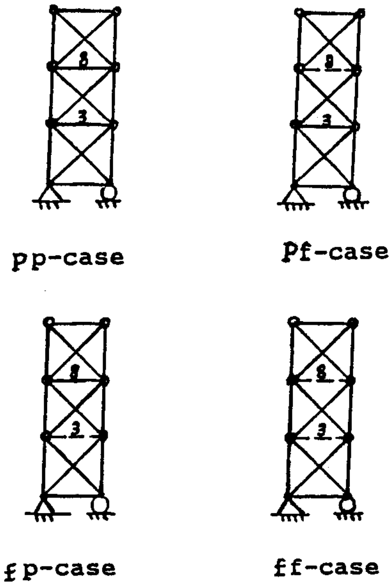

Fig. 16 Three-story truss with various pivotal bar conditions.

Combinational Pivotal Decomposition Method

For three or more story truss structures, the situation becomes more complicated. There are two or more separating key truss members needed to be recognized. A three-story truss system with 16 bars is shown in Fig. 16. There are more than 65,000 ($2^{16} = 65{,}536$) possible pass/fail damage modes. To handle such a large number of pass/fail structural configurations, a systematic and efficient methodology is urgently needed. In response to that need, the CPDM method is developed to circumvent most of difficulties. In Fig. 16, both bars 3 and 8 are given the status of pivotal members vs the single pivotal bar for a two-story truss structure discussed in the last section.

These two important bars can have four pass/fail combinations as *pp*, *pf*, *fp*, and *ff* shown in Fig. 16. The probability of survival for the *pp* case is rather conventional

$$({}_{pp}P_s)_{\text{syst}} = P_s^2(P_s^5 + 5P_s^4P_f)^2(P_s^4 + 4P_s^3P_f) \quad (21)$$

for *pf* and *fp* cases, the equations are the same

$$({}_{pf}P_s)_{\text{syst}} = ({}_{fp}P_s)_{\text{syst}} = P_fP_s(P_s^5 + 5P_s^4P_f)(P_s^9 + 9P_s^8P_f) \quad (22)$$

for the *ff* case

$$({}_{ff}P_s)_{\text{syst}} = P_f^2(P_s^{14} + 14P_s^{13}P_f) \quad (23)$$

thus, the system structural combat survivability becomes

$$(P_s)_{\text{syst}} = ({}_{pp}P_s)_{\text{syst}} + 2({}_{pf}P_s)_{\text{syst}} + ({}_{ff}P_s)_{\text{syst}} \quad (24)$$

The system survivability vs component survivability for the three-story truss structure is represented by Fig. 17. Furthermore, for the four-section truss structure as discussed in the foregoing bombardment analysis as shown in Fig. 7, there will be more than 2,000,000 (2^{21} = 2,097,152) possible damage modes. The three pivotal bars (3, 8, and 13) can have eight pass/fail combinations as *ppp*, *ppf*, *pff*, *fpp*, *fpf*, *fff*, *ffp*, and *pfp*. Moreover, for unequal component probability of survival for different category of truss bars as calculated by the foregoing Lee-Buffon bombardment analysis, say, due to probable different exposure configuration to gunfire, the cross bars 5, 6, 10, 11, 15, 16, 20, and 21 in Fig. 7 have a probability of survival different from the remainder of the four-section truss system. Again, by CPDM, the necessary equations to calculate the system survivability are given below, where P_{s2} and P_{f2} are for crossbars, while P_{s1} and P_{f1} are for the remainder of the bars in the truss system:

$$({}_{ppp}P_S)_{\text{syst}} = P_{S1}^3(P_{S1}^3P_{S2}^2 + 3P_{S1}^2P_{f1}P_{S2}^2 + 2P_{S1}^3P_{S2}P_{f2})^2(P_{S1}^2P_{S2}^2 + 2P_{S1}P_{f1}P_{S2}^2 + 2P_{S1}^2P_{S2}P_{f2})^2 \tag{25}$$

$$({}_{ppf}P_S)_{\text{syst}} = ({}_{fpp}P_S)_{\text{syst}} = P_{S1}^2P_{f1}(P_{S1}^3P_{S2}^2 + 3P_{S1}^2P_{f1}P_{S2}^2 + 2P_{S1}^3P_{S2}P_{f2})(P_{S1}^2P_{S2}^2 + 2P_{S1}P_{f1}P_{S2}^2 + 2P_{S1}^2P_{S2}P_{f2})(P_{S1}^5P_{S2}^4 + 5P_{S1}^4P_{f1}P_{S2}^4 + 4P_{S1}^4P_{S2}^3P_{f2}) \tag{26}$$

$$({}_{pff}P_S)_{\text{syst}} = ({}_{ffp}P_S)_{\text{syst}} = P_{S1}P_{f1}^2(P_{S1}^3P_{S2}^2 + 3P_{S1}^2P_{f1}P_{S2}^2 + 2P_{S1}^3P_{S2}P_{f2}) \times (P_{S1}^7P_{S2}^6 + 7P_{S1}^6P_{f1}P_{S2}^6 + 6P_{S1}^7P_{S2}^5P_{f2}) \tag{27}$$

$$({}_{fpf}P_S)_{\text{syst}} = P_{f1}^2P_{S1})(P_{S1}^5P_{S2}^4 + 5P_{S1}^4P_{f1}P_{S2}^4 + 4P_{S1}^5P_{S2}^3P_{f2})^2 \tag{28}$$

$$({}_{pfp}\mathrm{P}_S)_{\text{syst}} = P_{S1}^2P_{f1}(P_{S1}^3P_{S2}^2 + 3P_{S1}^2P_{f1}P_{S2}^2 + 2P_{S1}^3P_{S2}P_{f2})^2(P_{S1}^4P_{S2}^4 + 4P_{S1}^3P_{f1}P_{S2}^4 + 4P_{S1}^4P_{S2}^3P_{f2}) \tag{29}$$

$$({}_{fff}P_S)_{\text{syst}} = P_{f1}^3(P_{S1}^{10}P_{S2}^8 + 10P_{S1}^9P_{f1}P_{S2}^8 + 8P_{S1}^{10}P_{S2}^7P_{f2}) \tag{30}$$

Similarly, the system P_s can be obtained by adding these equations up appropriately. While substituting the probabilities of failure of Eq. (16) into Eqs. (25–30), the system combat survivability can be readily obtained as 0.8267. Therefore, the corresponding system failure probability is then 0.1733, which is much higher than any individual component failure probability. For a truss system with distinct component survivabilities, similar equations can also be similarly established. As mentioned before, this four-section truss has more than 2 million possible damage modes, this number, although it sounds large, is in fact minimal compared with a 46-bar electric power transmission truss tower with more than 70,368 billion (i.e., 2^{46}) possible damage modes.[21] Only 50 equations are needed to handle this gigantic number of damage modes efficiently by the CPDM method to obtain its system survivability, thus the power of this method can be easily seen. Furthermore, the route of approach of the fragments is not necessary to be assumed normal to the targeted truss. For a simple illustrative example, if the route is 30 deg off the normal line to the targeted plane as shown in Fig. 18, then the projection of the truss on the plane normal to the route of approaching fragments can be conveniently obtained first (Fig. 19), before the foregoing Lee-Buffon bombardment analysis method is applied to attain the survivability of each truss component.

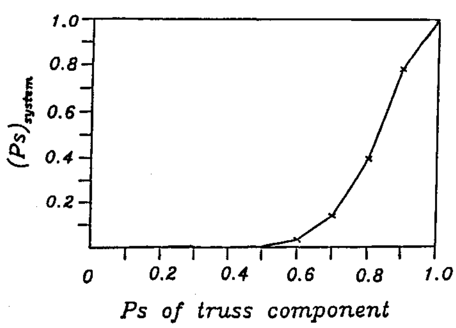

Fig. 17 System survivability vs component survivability for three-story truss.

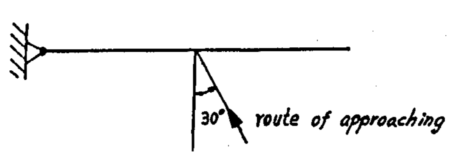

Fig. 18 Approaching route of fragment with 30-deg angle.

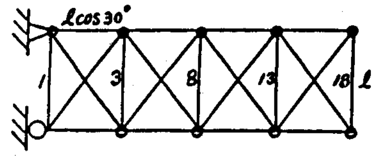

Fig. 19 Projected view of the truss with 30-deg angle.

Summary and Conclusions

The system combat survivability of structures is an essential issue of design analysis for many important structures, since system failures of structures are generally much more costly than uncollapsable nonsystem failures. Nevertheless, before such a truss system survivability can be analyzed, the bombardment survivability of each truss component should be predicted somehow. However, these are not easy tasks as reflected by current literature. Thus, in this article, an interesting Lee-Buffon bombardment analysis method is first developed in a decomposed and probabilistic sense. This interesting technique accommodates the bombardment analysis as a quasi-Buffon's falling-needle problem in a more realistic three-dimensional sense for each decomposed set of truss components. In conjunction, the Poisson model is delicately employed to model the fragment approaching situation and obtain the truss component survivability. Furthermore, the insurmountable analysis workload of damage modes, which defies all existing analysis techniques, has been overcome by the judicious combinational pivotal decomposition method.

In short, a variety of concepts and techniques have been ingeniously merged to constitute an interesting, inventive, and systematic bombardment survivability analysis methodology for two-dimensional, multiple-story, and statically indeterminate truss structures under firepower. The bombardment analysis method may also be applied for other truss structures,[21] such as strategically important truss bridges and major power transmission towers with various boundary conditions and/or nonparallel truss members. Additionally, with minor modification, the said bombardment analysis method may also be applied for space station truss structures subject to micrometeoroid or debris impacts.[10] The CPDM method can also be used to assess truss system survivability, not only due to combat damages, but also for cases due to structural aging, corrosion, fracture crack growth, fatigue, fire damage, ultralight/nuclear radiations, and other possible causes. Incidentally, the analysis objects of CPDM are not limited to truss system either, e.g., it has been successfully used for assessing the electric power system reliability of multiple-engined aircraft.[22] Moreover, the bombardment analysis method could be used for studying the truss structure size effect on its system survivability. Also, the CPDM method may be merged with other powerful techniques such as finite element method, random vibration analysis, and fatigue/fracture mechanics analyses to expand its range of usage.

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